Simple-minded estimate of the masses of baryons containing single heavy quarks *

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Abstract

The masses of the yet undiscovered baryons containing single c or b quarks are estimated from the known masses using the following rules: equal distances in mass between the isomultiplets forming sextets, equal mass differences between the corresponding spin one-half baryons containing c and b quarks, hyperfine splittings inversely proportional to the masses of the heavy quarks.

According to the familiar SU(4) classification of baryons (cf e.g. [1]) to every octet consisting of light (d, u, s) quarks only, corresponds, for every heavy flavour Q, a sexted and an antitriplet of spin one-half baryons containing each exactly one heavy quark Q. Q denotes here c or b, because the t-quark decays, usually into a W boson

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and a b-quark, before it has time to hadronize. In each baryon from the sextet the light two-quark system has spin one, which when combined with the spin one-half of the heavy quark can produce besides the spin one-half baryon also a spin three-halves baryon. This gives another sextet of baryons distinguished in the usual notation by an asterisk. The usual notation (cf e.g. [1]) is: Σ_Q and Σ_Q^* for the isotriplets from the sextets, Ξ'_Q and Ξ_Q^* for the isodoublets from the sextets, Ω and Ω^* for the isosinglets from the sextets, Λ_Q for the isosinglets from the antitriplets and Ξ_Q for the isodoublets from the antitriplets. There is some confusion about which spin one-half isodoublet should have the prime. We adhere here to the convention, which leaves the prime for the yet undiscovered isodoublet. One could object that $SU(4)$ symmetry is too badly broken to be a useful guide, but the same predictions follow from $SU(3)$ applied to the light diquark system. Altogether for $Q = c, b$ one expects 16 "ground state" baryons (isomultiplets) with single heavy quarks.

These baryons are in the process of being discovered. The 1994 Particle Data Group Tables [1] quote $\Lambda_c(2.2851 \pm 0.0006)$ (here and in the following the numbers in brackets following the symbol of the particle are its mass and the corresponding error, both in GeV), $\Xi_c(2.4677 \pm 0.0017)$ (for the isomultiplets we quote the average mass with the average error), $\Sigma_c(2.4531 \pm 0.0007)$ and $\Lambda_b(5.641 \pm 0.050)$. More recent results include $\Omega_c(2.7068 \pm 0.001)$ from the $WA89$ Collaboration reported at the Moriond (1995) Conference [2], a result from SKAT [3] $\Sigma_c^*(2.530 \pm 0.007)$, the CLEO result $\Xi_c^*(2.643 \pm 0.002)$ and the results reported at the Brussels EPS (1995) Conference [5] $\Lambda_b(5.638 \pm 0.016)$ from ALEPH, $\Sigma_b(m_{\Lambda_b} + 0.173 \pm 0.009)$ (here and in the following the statistical and the systematic error are added in quadrature) and $\Sigma_b^*(m_{\Lambda_b} + 0.229 \pm 0.009)$ from DELPHI.

In view of this progress on the experimental side, the theoretical activity, which has been going on for more than 20 years (cf e.g. [6], for a recent review cf. [7]), has recently significantly increased. Thus e.g. A. Martin and J. -M. Richard [8] quote ten predictions for the mass of the Ω_c ranging from 2.610 GeV to 2.783 GeV. A comparison with the experimental result (2.707 GeV) shows that all these predictions are good within 100 MeV, which is remarkable in view of the variety of approaches used. People have used potential models (cf e.g. [8]) and phenomenological fits inspired by such models (cf e.g. [9],[10]), suitably modified MIT bag models (cf e.g. [11]), suitably modified Skyrme

models (cf e.g. [12]), lattice methods (cf e.g. [13]) etc. This strongly suggests that the baryon spectrum is to a large extent defined by its general features common to most of the reasonable models. Finding the simple rules behind the models is of some interest. One can see, whether a model is really an improvement compared to the simple-minded version of the rules. One can also immediately distinguish the expected from the unexpected, when a new mass value is obtained from experiment.

In this note we try the following set of rules inspired by the heavy quark effective theory.

- The mass difference between any spin one-half b-baryon and the corresponding c-baryon is the same. Using the experimental masses of the baryons Λ_b and Λ_c we find for this difference $\delta_1 = (3.353 \pm 0.016)$ GeV.
- The isomultiplets in the sextets are equidistant in mass. In the light decuplet the corresponding mass differences are $m_{\Sigma^*} - m_\Delta = 153$Mev, $m_{\Xi^*} - m_{\Sigma^*} = 148$ MeV and $m_\Omega - m_{\Xi^*} = 139$ MeV. We expect a similar scatter of the differences — of a few MeV around an average — also for the sextets. Using the experimental masses of Σ_c and Ω_c we find for the mass difference between adjacent spin one-half isomultiplets from the sextets $\delta_2 = 0.127$[1] GeV. An immediate consequence of this assumption is that the known Ξ_Q particles are members of the antitriplets and not of the sextets.
- The hyperfine splittings i.e. the mass differences between the members of the spin one-half sextets and the corresponding members of the spin three-halves sextets depend only on the mass of the heavy quark Q and are inversely proportional to this mass. The second part of this assumption is a well-known leading term estimate from the heavy quark effective theory. Physically, it follows from the observation that the hyperfine splitting is proportional to the chromomagnetic moment of the heavy quark, which is inversely proportional to the mass of this quark. The first part is more model-dependent. For instance, Rosner [14] quotes a model, where the hyperfine splitting gets reduced by a factor of 0.84, when going from Σ_b to Ξ_b' and by a further factor

[1] Here and in the following the experimental uncertainties of the input are ignored, when they are much smaller than the other uncertainties.

of 0.81 when going from Ξ_b' to Ω_b. We have chosen equal splittings in order to keep the intervals between isomultiplets within the spin three-halves sextets equal. This choice is also supported by the data for mesons, where the hyperfine splittings for the D and D_s mesons are equal within a small experimental error. For $Q = c$ we find this splitting from the mass difference between the experimental mass of the Ξ_c^* and the interpolated mass of the Ξ_c':

$$M_{\Xi_c'} = \frac{1}{2}\left(M_{\Sigma_c} + M_{\Omega_c}\right). \tag{1}$$

This yields $\delta_{3c} = 0.063$ GeV for $Q = c$. For $Q = b$ we assume that the splitting is reduced by the ratio m_c/m_b, as suggested by the heavy quark approach. This factor can be estimated by a variety of methods, e.g. by comparing the hyperfine splittings for the B and D mesons. The result is about one third. Thus we put $\delta_{3b} = 0.021$ GeV.

On the whole, we use the experimental masses of six particles ($\Lambda_c, \Lambda_b, \Sigma_c, \Xi_c, \Omega_c$ and Ξ_c^*) and the ratio $\frac{m_c}{m_b}$ deduced from the meson data to fix our parameters. As a check we can find three other particle masses, for which preliminary experimental data is available:

$$\begin{aligned}
m_{\Sigma_c^*} &= m_{\Sigma_c} + \delta_{3c} = (2.516 \pm 0.010)\text{GeV}, & (2)\\
m_{\Sigma_b} &= m_{\Sigma_c} + \delta_1 = m_{\Lambda_b} + (0.168 \pm 0.005)\text{GeV}, & (3)\\
m_{\Sigma_b^*} &= m_{\Sigma_b} + \delta_{3b} = m_{\Lambda_b} + (0.189 \pm 0.020)\ \text{GeV}. & (4)
\end{aligned}$$

In order to eliminate the large uncertainty (± 0.016 GeV) in the parameter δ_1, we have introduced into the formulae for $Q = b$ the mass of the baryon Λ_b. Ignoring the theoretical uncertainties given above, our predictions deviate from the data by -2.0, -0.6, -4.4 standard deviations respectively. Since the data is preliminary, it is probably premature to draw conclusions from this comparison. Let us note, however, that if the experimental indication that the hyperfine splittings for $Q = b$ and $Q = c$ are similar were confirmed, this would be a serious difficulty for most present models.

The theoretical uncertainties have been estimated (very crudely!) as follows. The variations of hyperfine splittings quoted by Rosner [14] would increase the mass of Σ_c^* and decrease the mass of Ω_c^* by about

11 MeV each. This is believed to be an overestimate [14], therefore, 10 MeV was taken as a conservative estimate of this uncertainty. The uncertainty for Σ_b has been assumed to be comparable to the error on the mass of the B_s meson obtained from the formula $m_{B_s} = m_B + m_{D_s} - m_D$. This is smaller than the experimental uncertainty of about 6 MeV in m_{B_s}. Therefore, we guess an uncertainty of 5 MeV. For the $Q = b$, $J = \frac{3}{2}$ baryons, besides the usual uncertainty for $Q = b$ there is an additional uncertainty due to the uncertainty of the assumption of hyperfine splittings inversely proportional to the quark masses. This is large, since the preliminary DELPHI measurement of the Σ_b^*, Σ_b splitting gives 56 MeV, while we expect 21 MeV. We guess the total uncertainty of our prediction to be about 20 MeV which, when combined with the experimental error in quadrature, reduces the deviation for Σ_b^* to about 1.8 standard deviation.

For the yet undiscovered baryons our rules give:

$$m_{\Xi_c'} = (2.580 \pm 0.005)\ \text{GeV} \tag{5}$$

$$m_{\Omega_c^*} = m_{\Xi_c^*} + \delta_{3c} = (2.770 \pm 0.010)\ \text{GeV} \tag{6}$$

$$m_{\Xi_b} = m_{\Xi_c} + \delta_1 = m_{\Lambda_b} + (0.183 \pm 0.005)\text{GeV} \tag{7}$$

$$m_{\Xi_b'} = m_{\Sigma_c} + \delta_1 + \delta_2 = m_{\Lambda_b} + (0.295 \pm 0.010)\ \text{GeV} \tag{8}$$

$$m_{\Omega_b} = m_{\Sigma_b} + 2\delta_2 = m_{\Lambda_b} + (0.422 \pm 0.010)\ \text{GeV} \tag{9}$$

$$m_{\Xi_b^*} = m_{\Sigma_b} + \delta_2 + \delta_{3b} = m_{\Lambda_b} + (0.316 \pm 0.020)\ \text{GeV} \tag{10}$$

$$m_{\Omega_b^*} = m_{\Xi_b^*} + \delta_2 = m_{\Lambda_b} + (0.443 \pm 0.020)\text{GeV} \tag{11}$$

The uncertainty for Ξ_c' was estimated from the errors in the analogous predictions for the isomultiplets in the light decuplet: $m_{\Sigma^*} = \frac{1}{2}(m_\Delta + m_{\Xi^*})$ and $m_{\Xi^*} = \frac{1}{2}(m_{\Sigma^*} + m_\Omega)$. These errors are -2 MeV and -5 MeV respectively. Therefore, we estimate the error on $m_{\Xi_c'}$ as 5 MeV. For the mass of Ξ_b the uncertainty was assumed to be equal to that for the mass of Σ_b. For the heavier baryons Ξ_b' and Ω_b, this uncertainty has been doubled. The uncertainties for the remaining baryons have been discussed above.

It is instructive to compare our results with another purely phenomenological (no explicit dynamics) set of predictions given by R. Roncaglia et al. [10]. For $Q = c$ there is exact agreement for Ξ_c' and Ω_c^*, while for Σ_c^* our prediction is lower by 4 MeV. For $Q = b$, putting in our formulae $m_{\Lambda_b} = 5.638$ GeV, one finds: exact agreement for Ω_b,

a Ξ_b heavier by 11 MeV in our case and in all other cases heavier baryons in the approach od Roncaglia et al. For Σ_b^*, Ξ_b^*, Ω_b^* the differences are respectively 23 MeV, 26 MeV and 9 MeV. For Σ_b and Ξ_b' the differences are 14 MeV and 17 MeV. The discrepancies are well within the stated errors of the two approaches.

Let us conclude with a few comments. A calculation of the baryon masses with an uncertainty of 10 MeV or less from first principles would be much more interesting than the phenomenological fits. For the moment, however, it is not yet in sight. The present approach is purely phenomenological, but it is very simple and the physical assumptions are transparent. When the experimental data for the heavy baryon masses become available, it will be interesting to see, how it compares with the more sophisticated approaches. The predictions are based on seven free parameters taken from experiment: the masses of Λ_c, Ξ_c, Ξ_c^* and Λ_b and the parameters δ_2, δ_{3c}, δ_{3b} defined in the text. The mass of Λ_b may be traded for the parameter δ_1. This number of free parameters is not outrageous. A typical quark model would have used four quark masses (assuming $m_u = m_d$) and three parameters in the potential. The stated number of free parameters is somewhat a matter of taste. All the parameters are constrained. Moreover, some parameters may be presented as predictions of the theory. Suppose for example that our assumption that $\delta_{3c} = 3\delta_{3b}$ works. It is inspired by (but not rigorously derived from) the heavy quark approach and supported by the experimental results for the hyperfine splittings in the mesonic sector, therefore, one could present it as a theoretical result and reduce by one the number of free parameters. In fact, assessing the number of free parameters in various models one has to be very careful about such "hidden parameters". The most risky assumption in our approach is the estimate of δ_{3b}, which contradicts the preliminary result for the mass of Σ_b^*. If this experimental result is confirmed, one will have to increase δ_{3b} i.e. to shift up by equal amounts the expected masses of Σ_b^*, Ξ_b^* and Ω_b^*. Then, however, a theoretical problem arises: why the pattern of hyperfine energy splittings for heavy baryons is so different from that for the mesons, or equivalently why the ratios of the hyperfine splittings in baryons to those in mesons increases from below 0.5 for $Q = c$ to about 1.2 for $Q = b$? Let us repeat finally that our estimates of errors are very crude, based on uncertain analogies — though they are probably good enough to distinguish the more uncertain from the less uncertain predictions. To be sure, the theoretical

errors quoted are understood as analogues of one standard deviation and not as maximum conceivable errors.

References

[1] Particle Data Group, Phys. Rev. **D45** (1992) Part 2.

[2] F. Dropmann Rencontre de Moriond (1995) in print.

[3] V.A. Ammosov et al, *Pis'ma Zh. Eksper. Teor. Fiz.* **58** (1993) 241.

[4] P. Avery et al. *Phys. Rev. Letters***75** (1995) 4364.

[5] D. Bloch, Report at the EPS Conference Brussels (1995) in print.

[6] A. De Rujula, H. Georgi and S.L. Glashow, *Phys. Rev.* **D12** (1975) 147.

[7] J.G. Körner, M. Krämer and D. Pirjol, *Progress in Particle and Nuclear Physics* **33** (1994) 787.

[8] A. Martin and J. -M. Richard, *Phys. Letters* **B355** (1995)345.

[9] R. Roncaglia, A. Dzierba, D.B. Lichtenberg and E. Predazzi, *Phys. Rev.* **D51** (1995) 1248.

[10] R. Roncaglia, D.B. Lichtenberg and E.Predazzi, *Predicting the masses of baryons containing one or two heavy quarks* hep-ph/9502251.

[11] M. Sadzikowski, *Acta Phys. Pol.* **B24**(1993) 1121.

[12] M. Rho, D.O. Riska and N. N. Scoccola, *Phys. Letters* **B251** (1990) 597.

[13] K.C. Bowler et al, (UKQCD Collaboration) *Heave baryon spectroscopy from lattice* hep-lat/9601022.

[14] J.L. Rosner, *Phys. Rev.* **D52** (1995) 6461.